Note : cells with light green background have answers which match the text.

```
Clear["Global`*"]
```

1. Powers of i. Show that i^2=-1, i^3=-i, i^4=1, i^5=i, ... and $\frac{1}{i}$=-i, $\frac{1}{i^2} = -1$, $\frac{1}{i^3} = i$ ...

```
tab = Table[ⅈ^n, {n, -3, 5}]
```

$\{i, -1, -i, 1, i, -1, -i, 1, i\}$

```
tex = {"ⅈ^-3", "ⅈ^-2", "ⅈ^-1", "ⅈ^0", "ⅈ^1", "ⅈ^2", "ⅈ^3", "ⅈ^4", "ⅈ^5"}
```

$\{i^{-3}, i^{-2}, i^{-1}, i^0, i^1, i^2, i^3, i^4, i^5\}$

```
Grid[{tex, tab}, Frame → All]
```

i^{-3}	i^{-2}	i^{-1}	i^0	i^1	i^2	i^3	i^4	i^5
i	-1	$-i$	1	i	-1	$-i$	1	i

3. Division. Verify the calculation in (7). Apply (7) to $\frac{(26-18\,i)}{(6-2\,i)}$

The problem refers to numbered line (7) on p. 610 of text.

$$z = \frac{x_1 + i\, y_1}{x_2 + i\, y_2};$$

```
z1 = ComplexExpand[z]
```

$$\frac{x_1\, x_2}{x_2^2 + y_2^2} + \frac{y_1\, y_2}{x_2^2 + y_2^2} + i \left(\frac{x_2\, y_1}{x_2^2 + y_2^2} - \frac{x_1\, y_2}{x_2^2 + y_2^2}\right)$$

$$\text{lef} = \text{Together}\left[\frac{x_1\, x_2}{x_2^2 + y_2^2} + \frac{y_1\, y_2}{x_2^2 + y_2^2}\right]$$

$$\frac{x_1\, x_2 + y_1\, y_2}{x_2^2 + y_2^2}$$

$$\text{rig} = \text{Together}\left[i \left(\frac{x_2\, y_1}{x_2^2 + y_2^2} - \frac{x_1\, y_2}{x_2^2 + y_2^2}\right)\right]$$

$$\frac{i\,(x_2\, y_1 - x_1\, y_2)}{x_2^2 + y_2^2}$$

```
z2 = lef + rig
```

$$\frac{i\,(x_2\, y_1 - x_1\, y_2)}{x_2^2 + y_2^2} + \frac{x_1\, x_2 + y_1\, y_2}{x_2^2 + y_2^2}$$

$$z6 = \frac{(26 - 18\,i)}{(6 - 2\,i)}$$

$$\frac{24}{5} - \frac{7\,i}{5}$$

8 - 15 Complex Arithmetic
Let z_1=-2+11i, z_2=2 -i. Find:

```
Clear["Global`*"]
```

$z_1 = -2 + 11\,i$

$-2 + 11\,i$

$z_2 = 2 - i$

$2 - i$

9. $\text{Re}[z_1^2]$, $\text{Re}[z_1]^2$

$zr1 = \text{Re}[z_1^2]$

-117

$zr2 = \text{Re}[z_1]^2$

4

11. $\frac{(z_1 - z_2)^2}{16}$, $\left(\frac{z_1}{4} - \frac{z_2}{4}\right)^2$

$$\frac{(z_1 - z_2)^2}{16}$$

$-8 - 6\,i$

$$\left(\frac{z_1}{4} - \frac{z_2}{4}\right)^2$$

$-8 - 6\,i$

13. $\frac{(z_1 + z_2)}{(z_1 - z_2)}$, $z_1^2 - z_2^2$

$$\frac{(z_1 + z_2)}{(z_1 - z_2)}$$

$$\frac{3}{4} - \frac{i}{4}$$

$$z_1^2 - z_2^2$$

$$-120 - 40\,i$$

15. $4\,\frac{(z_1 + z_2)}{(z_1 - z_2)}$

$$4\,\frac{(z_1 + z_2)}{(z_1 - z_2)}$$

$$3 - i$$

16 - 20 Let z = x + I y. Find in terms of x and y :

```
Clear["Global`*"]
```

```
z = x + I y
```

$x + i\,y$

17. $\text{Re}[z^4] - \text{Re}[z^2]^2$

$$\text{ComplexExpand}\left[\text{Re}[z^4] - \text{Re}[z^2]^2\right]$$

$$-4\,x^2\,y^2$$

19. $\text{Re}\left[\frac{z}{\bar{z}}\right]$, $\text{Im}\left[\frac{z}{\bar{z}}\right]$

```
Clear["Global`*"]
```

$z = x + i\,y$

$x + i\,y$

$$aa = \text{Re}\left[\frac{z}{z^*}\right]$$

```
ComplexExpand[Re[(x + ⅈ y)/(Conjugate[x] - ⅈ Conjugate[y])]]
```

$$\frac{x^2}{x^2+y^2} - \frac{y^2}{x^2+y^2}$$

```
bb = ComplexExpand[Im[z/z*]]
```

$$\frac{2\,x\,y}{x^2+y^2}$$

A precaution about the symbol for complex conjugate. To make a typesettting compound like z^* using the exponent key '^', *looks* like a conjugate symbol but will not be treated as one. It seems necessary to do " z⋮conj ⋮ " , without the space of course, in order to get something that Mathematica recognizes.